Breakdown of the Bardeen-Stephen law for free flux flow in $Bi_2Sr_2CaCu_2O_{8+\delta}$

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Pulsed high-current experiments in single crystals of the high-T_c superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$ in a c-axis-directed magnetic field H reveal that the ab-face resistance in the free flux flow regime is a solely logarithmic function of H, devoid of any power-law component. Reanalysis of published data confirms this result and leads to empirical analytical forms for the ab plane and c axis resistivities, $\rho_{ab} \propto H^{3/4}$, which does not obey the expected Bardeen-Stephen result for free flux flow and $\rho_c \propto H^{-3/4} \log^2 H$.

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I. INTRODUCTION

Free flux flow (FFF) resistivity describes how fast the vortices in a type II superconductor move in the direction of an applied force.[1] It is a measure of how the momentum of the superfluid is transferred to the host lattice via quasiparticle excitations. The velocity-force relation expressed by the FFF resistivity has to be taken into account in interpreting any vortex transport, be it global or local. Since the primary source of dissipation in a type II superconductor in a magnetic field is vortex motion, FFF resistivity is also of great importance for technical applications.

A transport current exerts a Lorentz-Magnus force on a vortex and if other forces such as vortex-defect interaction (pinning) are negligible (i.e., the vortex motion is "free"), the velocity-force relation can be inferred from the resistivity ρ_{FFF}. The Bardeen-Stephen (BS) law[2] states that ρ_{FFF} is proportional to the density of vortices and therefore to the magnetic field H,

$$\rho_{FFF} = \gamma \rho_n (H/H_{c2})^{\beta}, \quad \beta = 1, \tag{1}$$

where ρ_n is the normal-state resistivity, H_{c2} is the upper critical field, and γ is a constant ≈ 1. This law has been experimentally established[3] for a number of conventional superconductors. In high-T_c materials the quasi-two-dimensional (2d) electronic structure, the nodes of the d-wave order parameter, and the structure of the vortex system may potentially influence FFF. The motion of 2d "pancake" vortices along their well-conducting ab plane leads to dissipation by flux flow; whereas in the poorly conducting c direction, dissipation is governed not by flux flow but by tunneling between weakly coupled ab planes. A quasiclassical calculation[4] suggests that the BS law is valid also in the d-wave case.

Experiments to test the validity of Eq. (1) in high-T_c superconductors—and especially in $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO), the model system of this study—are contradictory. Data on low-frequency transport in single crystals[5] and thin films[6,7] as well as microwave and millimeter wave impedance[8] are inconsistent with one another, agreeing only that *the BS law is not obeyed*. The resistivity often resembles a sublinear power law in the field. The situation is similar in other high-T_c materials with the notable exception of the results of Kunchur *et al.*[9,10] who find agreement with BS law in thin-film resistivity measurements in $YBa_2Cu_3O_7$ (YBCO). However, the resistance they measure does not saturate at high current, leading to an uncertainty in the value of ρ_{FFF} as discussed below.

The main difficulty in measuring the velocity-force relation is to take account of the pinning force, about which one has little detail. One way is to model pinning to interpret the surface impedance arising from local vortex motion. Another approach is to create experimental conditions where pinning is irrelevant as what occurs in a true (unpinned) vortex liquid. We extend our experiments to the nonohmic regime by applying sufficiently high current such that the pinning force is negligible compared to the Lorentz-Magnus force from the transport current.

II. EXPERIMENT

To measure the FFF resistivity we have made pulsed high-current transport measurements on single-crystal BSCCO with electrode contacts on the face parallel to the well-conducting ab planes in a c-axis-directed magnetic field. The *global* single-crystal resistance measured on the ab face in the ohmic regime and the asymptotic high-current differential resistance in the nonohmic regime show the same logarithmic magnetic-field dependence devoid of any power law. We combine this result with published data from other experiments[11,12] and set up empirical functional forms for the *local* resistivities ρ_{ab} and ρ_c valid over a broad range of temperature and field in the vortex liquid phase. The most striking and important of our conclusions is that for BSCCO $\beta = 3/4$ in Eq. (1).

We selected for experiment three single crystals from three different batches of BSCCO with typical dimensions $1 \times 0.5 \times 0.003$ mm^3, the shortest corresponding to the poorly conducting c axis. All were close to optimal doping with a resistance-determined critical temperature $T_c \approx 89$ K and transition width about 2 K in zero field; the diamagne-

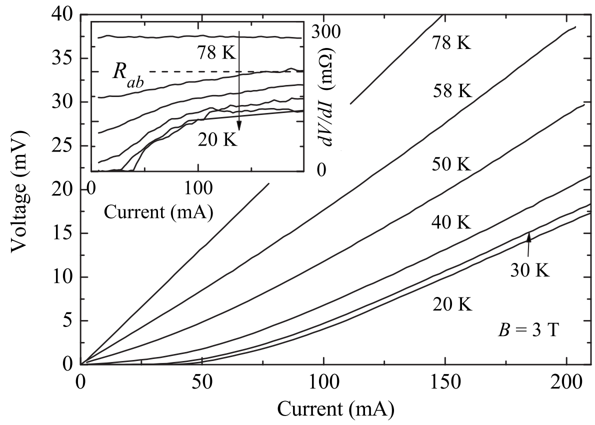

FIG. 1. Typical voltage-current characteristics at selected temperatures in $B=3$ T. Inset: Current dependence of differential resistance dV/dI at the same temperatures. At high currents dV/dI saturates at R_{ab}.

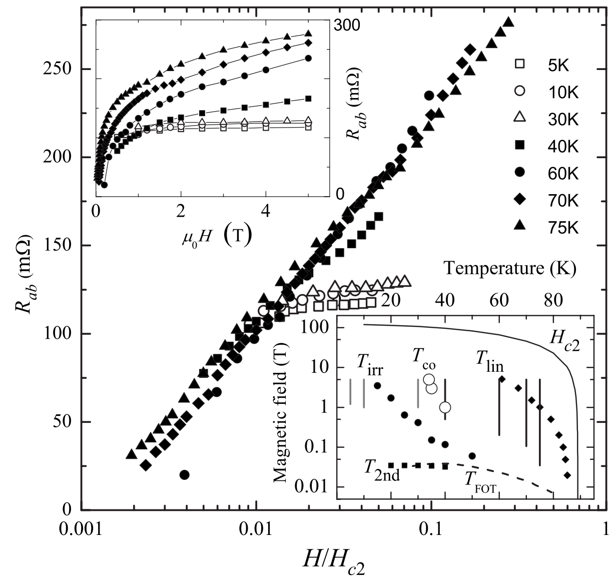

FIG. 2. High-current differential resistance R_{ab} at several temperatures as a function of the logarithm of magnetic field normalized to the upper critical field $H_{c2}(T)$. Upper inset: Same data on a linear field scale. Lower inset: Phase diagram measured on the same samples except for the FOT T_{FOT} taken from Ref. 15 for comparison. T_{irr} refers to magnetic irreversibility, T_{2nd} to the second magnetization peak, and T_{lin} to the beginning of linear *V-I*. $H_{c2} = 120[1-(T/T_c)^2]$ T. The vertical black (gray) lines show the range of full (open) symbols in the main panel. The open circle is the crossover in the field dependence of R_{ab}.

tism in a 1-mT field set in progressively below T_c to near 100% at low temperature.

The voltage-current (*V-I*) response was measured in the usual four-point configuration on an *ab* face in perpendicular magnetic field with two current contacts across the width near the ends and two-point voltage contacts near each edge of the same face. The contacts were made by bonding 25 μm gold wires with silver epoxy fired at 900 K in an oxygen atmosphere resulting in current contact resistances of less than 3 Ω. To avoid significant Joule heating, we employed short (≤ 50 μs) current pulses of isosceles triangular shape from 0.2 to 1 s intervals. Technical details and the issue of Joule heating are treated in Ref. 13 concluding that the temperature change in the area between the voltage contacts is negligible for the duration of the pulse.

III. RESULTS

Typical *V-I* characteristics at different temperatures in a field of 3 T are shown in Fig. 1. Above a temperature $T_{lin} < T_c$ the *V-I* curves are linear (see the lower inset of Fig. 2 for the field dependence of the characteristic temperatures). Below T_{lin} nonlinearity develops and the $I \to 0$ resistance decreases faster than exponentially until it becomes immeasurably small even with the most sensitive technique. At low temperature, dissipation sets in abruptly at a threshold current I_{th}; for higher temperatures a marked upturn in the *V-I* curve (a "knee") is seen at $I_k \lesssim I_{th}$. Throughout the nonlinear range the differential resistance increases with increasing current; for currents several times I_{th} or I_k, it saturates (becomes current independent) at a value $dV/dI = R_{ab}$ as shown in the inset of Fig. 1. Since I_{th} and I_k are hallmarks of depinning, the current-independent R_{ab} suggests that at these high-currents pinning is irrelevant and R_{ab} reflects FFF.[14] The focus of this paper is the behavior of R_{ab}.

The upper inset of Fig. 2 shows the field dependence of R_{ab} for the temperatures indicated in the phase diagram of the lower inset. The resistance is field and temperature independent at low temperature. With increasing temperature there is a crossover to a field-dependent behavior at T_{co} (open circles in the phase diagram) situated between the magnetic irreversibility line and T_{lin}. Having in mind that in the BS law the characteristic field is H_{c2}, we interpolate the upper critical field using the form $H_{c2} = (120\text{ T})[1-(T/T_c)^2]$ constructed from $dH_{c2}/dT|_{T_c} = -2.7$ T/K (Ref. 16) and use it to plot R_{ab} against H/H_{c2} on a logarithmic field scale in the main panel of Fig. 2. The high-temperature curves all collapse into a master curve representing a logarithmic field dependence

$$R_{ab}(H,T) = R_{ab}^n\{1 + \alpha \log[H/H_{c2}(T)]\}, \tag{2}$$

where R_{ab}^n is the zero-field normal resistance at T_c and α is a constant. This scaling only contains the temperature through $H_{c2}(T)$. Equation (2) provides an excellent description of all three samples; for the parameter α we find 0.16, 0.19, and 0.21, essentially the same values insensitive to the presumably different disorder between samples.

We emphasize that R_{ab} cannot be compared directly with the BS law because the strong anisotropy of the electronic properties makes the current distribution very inhomogeneous and R_{ab} reflects both *ab*-plane and *c*-axis properties. However, if the sample is thick in the *c* direction, a simple scaling argument for a current-independent local resistivity tensor yields $R_{ab} = A\sqrt{\rho_{ab}\rho_c}$ where A is a geometrical factor. This relation is valid in the linear region $T > T_{lin}(B)$ and also below T_{lin} if the current is sufficiently high such that the current density is well in the upper differentially linear por-

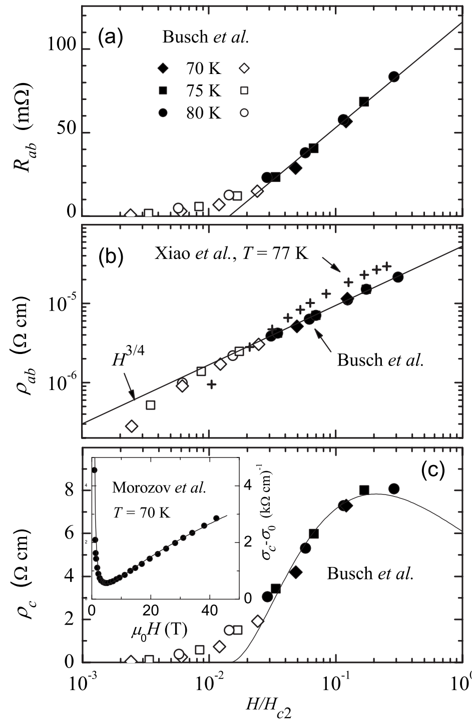

FIG. 3. (a) Single-crystal resistance R_{ab}, (b) ab-plane resistivity ρ_{ab}, and (c) c-axis resistivity ρ_c as a function of magnetic field normalized to the upper critical field H_{c2}. Data from Ref. 11. For full symbols $T > T_{\mathrm{lin}}$. Solid lines in panels (a)–(c) are fits to Eqs. (2)–(4), respectively. In panel (b) 77-K thin-film data from Ref. 7 are also shown by crosses. Inset of panel (c): c-axis conductivity data from Ref. 12. $\sigma_c = 1/\rho_c$ and σ_0 is a constant. The solid line is a fit to Eq. (3).

tion of the response near the top surface of the crystal.[14] The analysis and experimental checks of Ref. 11 indicate that with the sample size, shape, and contact geometry used in their and our single-crystal ab-plane studies in BSCCO, the thick sample limit provides a good description.

IV. DISCUSSION

Independent confirmation of our results emerges from analysis of other experiments. Although high-current data are absent, the fact that the V-I curves are linear for $T > T_{\mathrm{lin}}(H)$ allows comparison with low-current data in this temperature and field range. In Fig. 3(a) we show R_{ab} vs H curves extracted from the R_{ab} vs T data taken by Busch *et al.*[11] in different magnetic fields. Excellent agreement with Eq. (2) is seen for $T > T_{\mathrm{lin}}$ (full symbols in the figure) with $\alpha = 0.23$.

Busch *et al.*[11] were able to disentangle ρ_{ab} and ρ_c by using data from two additional contacts on the bottom of the crystal. ρ_{ab} and ρ_c results extracted from their work for a series of temperatures are shown as a function of H/H_{c2} in Figs. 3(b) and 3(c). In the temperature and field range $T > T_{\mathrm{lin}}(H)$ both quantities individually exhibit H/H_{c2} scaling. The in-plane resistivity does *not* agree with the $\beta = 1$ BS law of Eq. (1) but is well described by a $\beta = 3/4$ exponent (best fit $\beta = 0.75 \pm 0.02$). Having definite analytical forms for both R_{ab} and ρ_{ab}, we can use the relation $(R_{ab}/A)^2 = \rho_{ab}\rho_c$ to write an expression for the c-axis resistivity. In summary,

$$\rho_{ab} \equiv \rho_{\mathrm{FFF}} = \rho_{ab}^{n}(H/H_{c2})^{\beta}, \tag{3}$$

$$\rho_c = \rho_c^{n}(H/H_{c2})^{-\beta}[1 + \alpha \log(H/H_{c2})]^2,$$

$$\beta = 3/4, \quad \alpha = 0.2. \tag{4}$$

The prefactors ρ_{ab}^{n} and ρ_c^{n} are in good agreement with the respective normal resistivities at T_c. Although scaling the magnetic field with our estimate of $H_{c2}(T)$ using $H_{c2}(0) = 120$ T from Ref. 16 provides a good description of the data, single-crystal resistance shown in Fig. 2 together with Eq. (2) allows only an order-of-magnitude estimate of $H_{c2}(0)$ since the scaling field appears in the argument of a logarithmic function. Equation (3) and ρ_{ab} data from Fig. 3(b) permit a better estimate $H_{c2}(0) = 180 \pm 80$ T.

Are these forms corroborated by other types of measurement? In principle ρ_{ab} can be measured in thin films where the current density is expected to be homogeneous. In Fig. 3(b) we show the 77-K thin-film resistivity obtained by digitizing the V-I curves in Ref. 7. Data above about 1 T are reasonably well described by a $H^{3/4}$ dependence, but a closer look reveals that the log ρ_{ab} vs log H curves are concave from below at every H, i.e., there is a systematic deviation from the power law. This "logarithmlike" (but not logarithmic) dependence is shared with other thin-film results,[6] but there are significant quantitative differences between data measured by different groups. A possible reason is that macroscopic defects such as steps on the surface or mosaic boundaries force c-axis currents and the measured resistance is a sample-dependent combination of ρ_{ab} and ρ_c.

The expression for ρ_c reproduces well the maximum [at $H_{\max} = H_{c2}\exp(8/3 - 1/\alpha) \sim 0.1H_{c2}$] observed in the high-field c-axis magnetoresistance,[12] and the overall field dependence of these independently measured data is very well described by Eq. (4). We demonstrate this in Fig. 3(c) where we plot 70 K data for $\sigma_c(B) - \sigma_0$ from Fig. 5 of Ref. 12 where $\sigma_c = 1/\rho_c$ and the constant σ_0 is interpreted as the zero-field quasiparticle conductivity. Using our estimate of $H_{c2}(T) \approx 46$ T for $T = 70$ K, Eq. (4) fits the measured data with the parameters $\sigma_c^{n} = 1/\rho_c^{n} = 6.6$ (kΩ cm)$^{-1}$, $\sigma_0 = 3.6$ (kΩ cm)$^{-1}$, and $\alpha = 0.20$ to obtain the curve indicated by the continuous line in the figure. The value of α is in excellent agreement with that inferred from R_{ab} measurements. It should be pointed out, however, that σ_0 is significantly smaller than the value $\sigma_0 \approx 8$ (kΩ cm)$^{-1}$ inferred in Ref. 12. In terms of resistivities, this means that ρ_c decreases more slowly in high fields than described by Eq. (4) with $\beta = 3/4$ and is in fact best described with an exponent $\beta = 0.51$.

In the high-current limit the same form for $R_{ab}(H)$ also holds below $T_{\mathrm{lin}}(H)$ where the V-I curves are nonlinear. Since no change in the behavior of R_{ab} is observed when the $T_{\mathrm{lin}}(H)$ line is crossed, it is reasonable to assume the same for ρ_{ab} and ρ_c. In the low-field direction a lower limit of the validity of Eq. (2) is the zero of the equation at $H_0/H_{c2} = e^{-1/\alpha} \sim 10^{-3} - 10^{-2}$, higher but in the order of the first-order transition (FOT) in the static vortex system. In the high-field direction Eq. (2) is valid up to the highest field $\approx 0.3H_{c2}$ we investigated.

The temperature T_{co} of the crossover from R_{ab}=const to $R_{ab} \propto \log H$ is distinctly higher than the onset of magnetic irreversibility at T_{irr} and also above the vortex glass transition $T_g \approx T_{irr}$ inferred from scaling analysis[17] of the *V-I* curves. On the other hand, no change in the behavior of R_{ab} is observed when the $T_{irr}(H)$ and $T_g(H)$ lines are crossed. This suggests that because the pinning potential is smoothed at high velocities, the phase diagram of the far-from-equilibrium *dynamic* vortex system[18] is different from that of the unperturbed thermodynamic phases. Since R_{ab} behaves the same in the pinned ($T < T_{lin}$) and unpinned ($T > T_{lin}$) liquid phases, we propose that the unpinned phase, otherwise observed only above T_{lin}, may be restored in the range $T_{co} < T < T_{lin}$. Then T_{co} may approximate the melting transition in a hypothetical defect-free crystal.

Our most robust finding, invariably observed not only in our three batches but also in the data of Ref. 11, is the logarithmic field dependence of the high-current single-crystal resistance R_{ab}. Although a power of H factor is expected both in ρ_{ab} (Ref. 1) and ρ_c,[19] no such factor is present in $R_{ab} \propto \sqrt{\rho_{ab}\rho_c}$. The most likely reason is that the power-law factors in ρ_{ab} and ρ_c cancel (exponents 3/4 and −3/4 in our analysis). The cancellation is very accurate; we estimate that a power-law factor with exponent as low as 0.1 could be observed in our R_{ab} data. Moreover, because we find no logarithmic correction to ρ_{ab}, the logarithmic dependence of R_{ab} is carried by ρ_c.

Arguing that both ρ_{ab} and σ_c are proportional to the quasiparticle density of states at the Fermi level $N(0)$, it cancels in the product $\rho_{ab}\rho_c$. In conventional superconductors, $N(0)$ is proportional to the number of vortices and therefore to H, leading to the H-linear resistivity of the BS law. In nodal gap superconductors near-nodal quasiparticles lead to a sublinear dependence; for line nodes $N(0) \propto H^{1/2}$.[20] Although delocalized near-nodal quasiparticles are not expected to contribute significantly to ρ_{FFF} because of the weak spectral flow force[4] they experience, the result may be different in the diffusive limit in the liquid phase. A possible reason for the $\beta = 3/4$ exponent is the different structure factors of the solid and liquid phases.

A mechanism common to both in-plane and interplane dissipation may lead to cancellation of the power-law factors and influence the value of β. Simultaneous in-plane and interplane phase shifts are an example of such a mechanism; however, this effect is expected to lead to $\rho_c \propto \rho_{ab}$ as suggested[21] for low currents in the pinned liquid phase, in disagreement with our results.

It is not yet clear if these findings can be generalized to other high-T_c superconductors. A reanalysis of Kunchur *et al.*'s[9] data in YBCO in terms of dV/dI instead of V/I (Ref. 14) yields results compatible with $\beta = 3/4$. In-plane resistivity from microwave surface impedance[22] above the FOT in YBCO is best described by $\beta = 0.77 \pm 0.06$.

V. CONCLUSION

In conclusion, we have set up empirical rules for the analytical form of single-crystal resistance as well as for the *ab*-plane and *c*-axis resistivities in the high-current FFF limit in the vortex liquid state of BSCCO, valid over a broad range of temperature and field. Both the logarithmic field dependence of the single-crystal resistance and the 3/4-power law in the *ab*-plane FFF resistance are in disagreement with the current theoretical understanding of high-T_c superconductors.

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